Isotropization at finite coupling

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Intro: QGP and holography

- AdS/CFT is a great arena to study strongly coupled CFT
- Study time-dependent evolution of QGP by solving classical gravity in AdS
- Remarkable tool: characteristic formulation of GR [Chesler, Yaffe]. Consider null foliation of space-time, eoms acquire nested form. Full time-evolution determined by solving – nested – linear ODE's.
- Einstein gravity corresponds to infinite coupling. Capture finite coupling corrections by considering higher curvature terms. Expect Gauss-Bonnet gravity to be a good toy model of these corrections
- Today: effect of finite coupling (Gauss-Bonnet) on isotropization.

Outline

- Isotropization in characteristic formulation
- Corrections from Gauss-Bonnet
- Main lessons
- Summary and outlook

Holographic Isotropization

Consider spatially homogeneous but unisotropic plasma in the initial state. This can be described in the dual gravity theory by

$$ds^2 = -2A(r,t)dt^2 + 2dtdr + \Sigma(r,t)^2(e^{B(r,t)}dx_\perp^2 + e^{-2B(r,t)}dx_\parallel^2)$$

Choose asymptotically AdS bc's and prescribe $B(r, t = 0)$.

$B \to 0$ at $t \to \infty$, when this happens the system has become isotropic.

Ward identities imply that the energy density is constant. The interesting observable is Δp

$$\Delta p = T^{zz} - \frac{1}{2}(T^{xx} + T^{yy}) \propto \partial_u(u^{-3}B)\big|_{u=0}, \qquad u = 1/r$$

No hydro modes excited (no spatial dependence).

Isotropization in characteristic formulation

Characteristic formulation: metric written in null foliation. Adapted to this, write the time derivatives in terms of

$$d_+ = \partial_t + A\partial_r$$

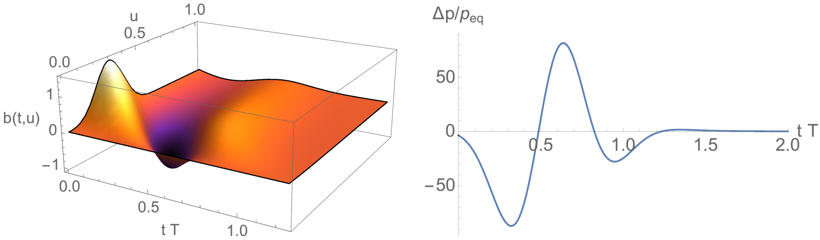
This allows us to write the eoms in a nested pattern

$$\begin{aligned}
0 &= \Sigma'' + \frac{1}{2}\Sigma\,(B')^2\,, \\
0 &= (d_+\Sigma)' + 2(d_+\Sigma)\frac{\Sigma'}{\Sigma} - 2\Sigma\,, \\
0 &= (d_+B)'\,\Sigma + \frac{3}{2}(d_+B)\Sigma' + \frac{3}{2}(d_+\Sigma)B'\,, \\
0 &= A'' - 6(d_+\Sigma)\frac{\Sigma'}{\Sigma^2} + \frac{3}{2}(d_+B)B' + 2\,,
\end{aligned}$$

Provided $B(r, t = 0)$ and asymptotic bc's, solution is determined uniquely (solve linear ODE's and integrate in time at each step)

Isotropization in characteristic formulation 2: results

Consider Gaussian profile $B(u, t = 0) = \beta u^4 \exp(-(u - u_0)^2/w^2)$



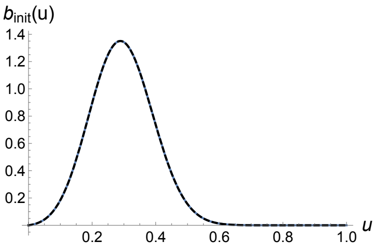
- System isotropizes quickly $b(t_{is0}) \ll 1$ for $t_{iso} \sim 1/T$
- For generic IC, evolution well described by QNM [Heller, Mateos, van der Schee, Triana]

Isotropization in characteristic formulation 3: QNM match

Expand initial condition in QNM

$$b(u,0) = \sum_{i=1}^{N_{QNM}} C_i \phi_i(u), \quad \text{lin. thy. predicts} \quad b(u,t) = \sum_{i=1}^{N_{QNM}} C_i \phi_i(u) e^{-i\omega_i t}$$

Do regression to determine the C_i's and compare. For $N_{QNM} = 10$



[Heller, Mateos, van der Schee, Triana] found that this holds true quite generically (avoid caustics, better for some IC than others).

Finite coupling corrections

In AdS/CFT, α' corrections to SUGRA map to finite 't Hooft coupling. In IIB, first correction is $\sim \gamma R^4$, and can be treated perturbatively

$$\frac{\eta}{s} = \frac{1}{4\pi}(1 + 135\gamma)$$

Toy model: consider $\sim \lambda_{GB} R^2$ correction which preserves second order eoms, namely Gauss-Bonnet gravity. Similar behaviour of the viscosity for $\lambda_{GB} < 0$

$$\left.\frac{\eta}{s}\right|_{GB} = \frac{1}{4\pi}(1 - 4\lambda_{GB})$$

Also QNM_{GB} behave similarly to QNM_{R^4} for $\lambda_{GB} < 0$ [Grozdanov, Kaplis, Starinets]

For computational simplicity, we consider Gauss-Bonnet here

Gauss-Bonnet gravity 2

$$S_{GB} = \int d^5x\sqrt{-g}\left(R + 12 + \frac{\lambda_{GB}}{2}\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right)\right)$$

- Leads to second order eoms

$$E_0 + \lambda_{GB}E_{GB} = 0$$

- Known stationary BH solution

$$ds^2 = \frac{dr^2}{f(r)} - f(r)dt^2 + r^2 d\vec{x}^2, \quad f = \frac{r^2}{2\lambda_{GB}}\left(1 - \sqrt{1 - 4\lambda_{GB}(1 - r_H^4/r^4)}\right)$$

- Gravitational QNM have been computed [GKS]
- Holographic renormalization is known. Can compute $T^{\mu\nu}$. [Brihaye, Radu]

Gauss-Bonnet gravity 2, perturbation theory

Want to study isotropization in this modified theory. Using the same d_+, not obvious that the nested structure is preserved.

Treat the problem perturbatively

$$g = g_0 + \lambda_{GB}\delta g, \quad \Rightarrow \qquad E_{0,\mathrm{lin}}[\delta g] = -E_{GB}[g_0]$$

Eoms are by construction linear (and nested!)

Solve for the linear perturbation using:

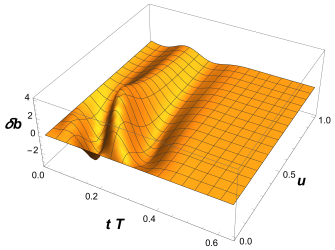
- Initial condition: keep anisotropy fixed
- Boundary condition: keep energy fixed

Given a background, find unique linearized solution.

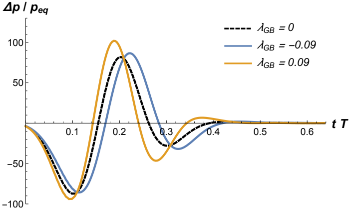
Isotropization for Gauss-Bonnet, results

Book-keeping $b = b_0 + \lambda_{GB}\delta b$,

$$\Delta p = \Delta p_0 + \lambda_{GB}\delta(\Delta p), \qquad \delta(\Delta p) = -3\partial_u\left(\delta b + \frac{1}{2}b_0\right)|_{u=0}$$



(a)

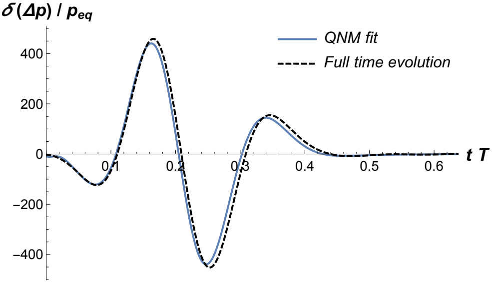


(b)

Note: shift of Δp given by the sign λ_{GB}. Observe qualitatively similar behaviour for all IC's. True in general?

Isotropization for Gauss-Bonnet, match QNM

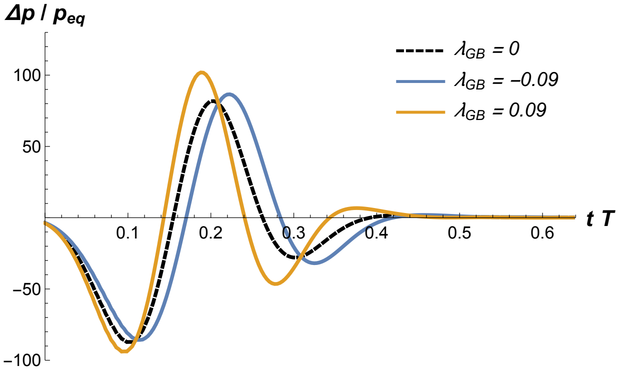
Good agreement with QNM, as in the $\lambda_{GB} = 0$ case.



We can use this simplification to provide an argument for the observed shift.

Shift in Δp using QNM

Want to argue that Δp is "approximately shifted" from Δp_0, for all initial conditions.



More precisely, $\Delta p'(t)$ and $\delta(\Delta p)(t)$ have the same sign for $t > \tilde{t}$ with $\tilde{t}$ small.

Shift in Δp using QNM, cont'd

From the QNM expansion,

$$b(u,t) = \mathrm{Re}\sum_i C_i \phi_i(u) e^{-i\omega_i t}$$

Expand

$$\phi_i = \phi_i^{(0)} + \lambda_{GB}\,\delta\phi_i, \quad C_i = C_i^{(0)} + \lambda_{GB}\,\delta C_i, \quad \omega_i = \omega_i^{(0)} + \lambda_{GB}\,\delta\omega_i$$

and obtain

$$\delta b(u,t) = \sum_i e^{-i\omega_i^{(0)} t}(\delta C_i \phi_i^{(0)}(u) + C_i \delta\phi_i^{(0)}(u) - it\delta\omega_i C_i^{(0)} \phi_i^{(0)}(u))$$

$$\delta(\Delta p)(t) \sim \partial_u \delta b(0,t)$$

$\delta b(u,0) = 0$ relates δC_i with C_i via regression.

Shift in Δp using QNM, cont'd

Key point: relation between δC_i with C_i is given by a universal (IC-independent) matrix. This allows us to write

$$\delta(\Delta p) = \sum_{k=1}^{2N_{QNM}} \mathcal{C}_k \mathcal{F}_k(t), \qquad \mathcal{C}_k = \{\mathrm{Re}\, C_i, \mathrm{Im}\, C_i\}$$

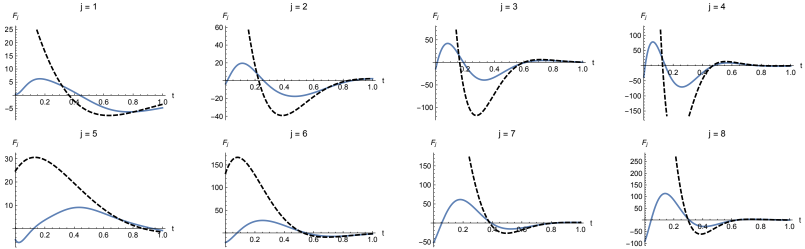
where $\mathcal{F}_k(t)$ are universal. Also, trivially,

$$\Delta p_0 = \sum_{k=1}^{2N_{QNM}} \mathcal{C}_k \mathcal{F}_k^{(0)}(t)$$

For our purposes, it is sufficient that $\mathcal{F}_k^{(0)}(t)$ and $\partial_t \mathcal{F}_k(t)$, have the same sign. These are independent of the IC's, and we can easily check $\mathcal{F}_k^{(0)}(t)\partial_t \mathcal{F}_k(t) > 0$ using background quantities.

Shift in Δp using QNM, cont'd

$$\delta(\Delta p)'(t) \qquad \text{same sign as} \qquad \Delta p_0$$



"Q.E.D"

Summary

- We have extended the analysis of far-from-equilibrium processes to higher derivative holography
- Time evolution in Gauss-Bonnet is in good agreement with QNM picture
- At least in the linear regime, we find that isotropization time grows with η/s
  - Shift in $\Delta p/p$, controlled by the sign of λ_{GB}. Provided a semi-analytic argument
- Contrast with bona-fide α' correction. Same structure, just a bit messier. Expect same behaviour for $\lambda_{GB} < 0$